Performance Based Learning and Assessment Task

Running Laps During Soccer Practice

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
Students will use their geometric knowledge in a real world situation by deciding what path to run during laps in their next soccer practice. Students will need to consider whether the dimensions of the soccer field matter in this problem or whether perimeter and the Pythagorean Theorem are most important.

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Measurement & Geometry

V. OBJECTIVES:
SOLs: G.8, G.9

VI. REFERENCE/RESOURCE MATERIALS:
Students will use: Measuring Wheel, Internet (to research the dimensions of the soccer field), TI-83 Plus (or higher) Graphing Calculator, Computer, Pencil, Paper, Assessment Rubric, Copy of Performance Task w/ Higher Order Questions, Word Processing Software (i.e Microsoft Word or Google Docs)

VII. PRIMARY ASSESSMENT STRATEGIES:
Rubric and assessment lists for tasks (includes the mathematics content, process skills, and requirements for the finished product)

VIII. EVALUATION CRITERIA:
Students will be evaluated on their successful completion of the activity, their short class presentation, and by their ability to answer the higher order questions posed to them by the teacher

IX. INSTRUCTIONAL TIME:
Two ninety-minute class sessions

Running Laps During Soccer Practice/Task 2

Strand

Measurement & Geometry

Mathematical Objective(s)

Students will be able to: 1)use the properties of rectangles to solve real-world problems, 2)find the perimeter of the soccer field, 3)T.S.W solve real-world problems using properties of right triangles and the Pythagorean Theorem (finding the diagonal distance of the soccer field), 4)determine which path they would like to use to run 20 laps during soccer path and explain their reasoning to their classmates in the form of a 2-3 minute presentation, 5)successfully answer a variety of higher order questions to demonstrate their understanding of proportional reasoning, unit and time conversions, distance, and speed calculations

Related SOLs

- SOL G.8 (The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry)
- SOL G.9 (The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.)

NCTM Standards

- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Understand the Pythagorean Theorem as it relates to right triangles.
- Apply the Pythagorean Theorem to real world situations
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest.
- Make decisions about units and scales that are appropriate.
- Convert between different systems of measurement.
- Convert commonly used measurements to equivalent units within the same system.
- Choose the appropriate customary or metric unit for length, area, perimeter, surface area or volume to measure an object.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Apply formulas for finding the perimeter and area of squares and rectangles.
- Organize their mathematical thinking through discussion with peers.
- Communicate their thinking clearly to teacher and peers.

- Analyze and evaluate the mathematical thinking and strategies of their partners.
- Use the language of mathematics to express mathematical ideas precisely.
- Create and use representations to record and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations.
- Use representations to model and interpret physical and mathematical phenomena.

Materials/Resources

Students will need: Measuring Wheel, Internet (to research the dimensions of the soccer field), TI-83 Plus (or higher) Graphing Calculator, Computer, Pencil, Paper, Assessment Rubric, Copy of Performance Task w/ Higher Order Questions, Word Processing Software (i.e Microsoft Word or Google Docs), *Extension Activities Sheet for Further Extension of the Lesson (Relates this Activity to the Triangle Inequality Theorem (a topic that students have not learned yet), Website: http://en.wikipedia.org/wiki/Triangle_inequality

Assumption of Prior Knowledge

- Rectangle characteristics/properties
- How to calculate the perimeter of a rectangle
- Knowledge of the Pythagorean Theorem
- How to do unit and time conversions
- Experience with using Word Processing Software

Introduction: Setting Up the Mathematical Task

The mathematical goal of this activity is for students to choose which path they would like to use to run 20 laps during soccer practice by using mathematics (computations, unit and time conversions), logical thinking, research, and their knowledge of perimeter and the Pythagorean Theorem . The goal is also to help the students to use mathematical reasoning to help them solve this real-world problem that arose from their coach giving the athletes options. The option the students choose will depend on the student's view on running and perspective on its purpose during practice. We will find out if students favor a shorter or longer path. Students may use word processing software to type out their presentation (bonus points). Nevertheless, students will gain valuable skills as they gather information and communicate their mathematical ideas.

Teachers should address the task as follows:

- Use the students complaining about having to run 20 laps during soccer practice and the coach's statement about choosing their running path as a teachable moment.

- Students can pick a partner or the teacher can create the groups
- Teacher will give an outline of the performance assessment task as students look at the typed version that is passed out to them
- Have students brainstorm with their partner about what information they will need to make their choice (possible responses: dimensions of the soccer field, shape of the soccer field, distance around the soccer field, distance from corner to corner, whether they should keep the units in yards or change to feet or meters)
- The teacher will pass out a rubric and assessment list that students will use as they work in groups of 2 as they work to come up with a solution to this real-world problem. The teacher will reinforce the idea that he or she is going to be in the role of facilitator and that the students will be responsible for developing a plan and implementing that plan to come up with a solution. Students will have to use Mathematics to solve the problem, but will be given access to a computer and the Internet to help them.

Student Exploration

Student Teacher Actions:

Students will first need to research the dimensions of a soccer field. They will have to decide which unit of measurement their group will use in their calculations. As students explore how long it will take them to complete the 20 laps using each of the two different paths, students will be doing additional conversions with time and distance. As students answer the higher order questions from the teacher and explore their own running paces / times, they will delve into areas they may have never explored or thought of before. The teacher will be there for support and as a facilitator to help the students stay on task. He or she will help guide the students and use a variety of questions as students participate in this real-world exploration.

Monitoring Student Responses

- I expect students to work together as they communicate their thinking and their new knowledge with each other and with the teacher.
- The teacher will help students with any clarification needs and assist students who are having difficulties by helping them connect their previous experiences to these new ones.
- Students who are ready to prepare their class presentation, can go ahead and move to the computer to begin typing out key ideas/information (they will earn bonus points).
- Closure will involve the students sharing their choices and mathematical reasons with the class in the form of a 2-3 minute presentation, the teacher asking a variety of questions, and the class going out to the soccer field with a measuring wheel to measure the actual dimensions and comparing their results to what they have found. Students will also reflect on whether these measures make them want to revisit their previous results or whether it confirms their previous results.

Assessment List and Benchmarks

Class worksheets, benchmarks, and rubrics are attached.

Rubric

			Earned Assessment	
#	**Element**	**Point Value**	**Self**	**Teacher**
1	Student participated in discussions with his/her partner and the class by speaking or asking a question at least three times (recorded with an observation checklist by instructor).	2		
2	Student developed a plan for solving the real-world problem.	2		
3	Student researched and found realistic dimensions for a soccer field.	2		
4	Student recorded measurement findings while completing the activity and used proper mathematics to verify solutions.	2		
5	Student selected which path he or she would take to run 20 laps during soccer path (as evidenced during the presentation).	2		
6	Student explained their mathematical reasoning behind their path selection (as evidenced during the presentation).	2		
7	Student actively participated in a 2-3 minute class presentation.	2		
8	Student answered all of the higher-order questions posed to him or her by the teacher after the presentation.	2		
9	Student demonstrates use of technology in their solution or final product.	2		
10	Student's presentation is well organized.	2		
11	Student's work from the post-presentation questions is neat.	2		
12	Student can explain their reasoning for their ideas, formulas, and work shown from activity.	2		
13	Student uses Word Processing Software to type presentation key ideas / information (**Bonus Points!)**	2		

Performance Based Assessment Task

Name(s): _____

_____

Date: _____

Geometry Teacher: _____

Running Laps During Soccer Practice

On Monday, the soccer coach tells the players that she is going to change things up a little bit on Thursday during practice. Instead of running 10 laps, the players will have to choose between running 20 laps around the outside of the field (the typical path) or run 20 inside laps (a full lap = going to the opposite corner and back). Before Thursday's practice begins, the student athletes have to tell the coach their decisions. This change upsets the players and they can't stop talking about it with their friends, even in the middle of Geometry class. These players need help making a mathematically informed decision. You will work together with a partner to brainstorm, make a plan to solve this real-world problem, outline the necessary steps, use logical thinking, research, mathematical reasoning, prior knowledge of applicable mathematical concepts, and be able to clearly explain your choice and the reasons behind your choice during a 2-3 minute class presentation. As you brainstorm, consider whether the dimensions of the soccer field matter in this problem. If you type the key concepts / information used for your presentation, you will earn bonus points!

Name: _____

Date: _____

Geometry Teacher: _____

Follow-up / Extension Activities (*Each Student Completes This!*)

Post-Presentation Questions *(You Must Justify Your Answers!)*

Sample Questions:

1. What are the dimensions of our school soccer field? How do you know?

2. Do the dimensions of the soccer field matter in this problem? If not, why?

3. If you normally run 10 laps in 25 minutes, how long will it take you to run 20 laps?

4. How far (in yards, feet, or meters) are you running each minute on average?

5. What is your average running speed?

6. If you maintain this pace, will it take you longer to run the 20 laps around the soccer field or 20 laps from corner to corner? How much longer?

7. Is it further to run 20 laps around the outside of the soccer field or 20 laps from corner to corner (along the diagonal)? How much further?

8. If the coach really wants some players to run "outside" laps and other players to run "inside" laps, calculate how many fewer "outside" laps should be run so that everyone finishes at roughly the same time.

9. Determine if the coach is spending too much time on having the players run 20 laps instead of having them practice soccer drills. If the soccer team normally practices for 90 minutes with a 5 minute water break included, what percent of practice is spent running these 20 laps? Make sure you calculate the percentages for both paths.

Extension Activities

10. Now, that we have walked through these steps, use your own personal running data or estimated times to answer questions 2-5 again. Show your work in the box below and place your responses to these questions in order on the lines below:

    a. _____

    b. _____

    c. _____

Work:

11. Our class will go outside to the soccer field and use a measuring wheel to verify your measurements. You will measure the actual dimensions and answer the questions below:
    a. Compare the measuring wheel results to the information you had found on the Internet.

    b. Do these recent measurements make you want to revisit your previous conclusions or do they confirm your previous results?

Additional Extension Activities to this Lesson

(Relates to the Triangle Inequality Theorem)

GEOMETRY

SOL G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will
a) order the sides by length, given the angle measures;
b) order the angles by degree measure, given the side lengths;
c) determine whether a triangle exists; and
d) determine the range in which the length of the third side must lie.
These concepts will be considered in the context of real-world situations.

In the soccer practice performance based assessment task, students use the Pythagorean Theorem to help them find the length of the diagonal of a soccer field because this is what they have experience working with. They have not yet explored properties that apply to all triangles, such as the Triangle Inequality Theorem. But, this performance based task can be a great transitional tool to students understanding why the sum of the two sides of any triangle has to be greater than the third side. Here is a website that can be used as a resource for both the teacher and the students: http://en.wikipedia.org/wiki/Triangle_inequality.

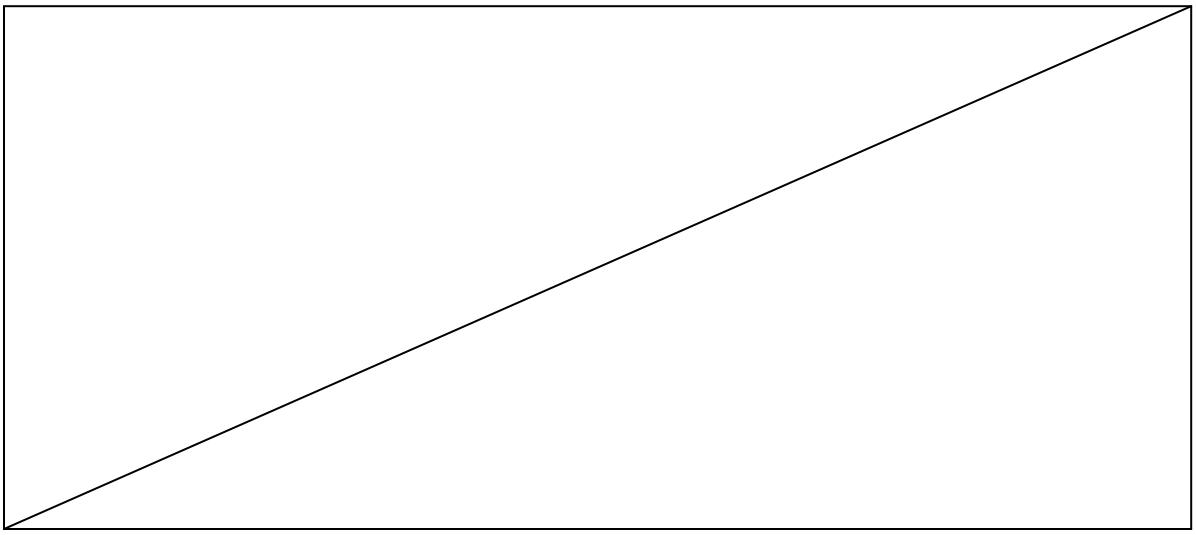
How could this potentially transform this Performance Based Assessment Task?

Since the two different groups of players are doing 20 laps each, students could examine what happens when the runners do 1 lap or even what happens when they do ½ lap each. This might help with congestion because the "outside" group runs half-way around the field while the "inside" group runs along the diagonal. This is a real-world example where we are just comparing the length of the two legs of a right triangle to the length of the hypotenuse. The sum of the two legs will always be longer, due to the Triangle Inequality Theorem, which applies for any triangle on the plane. This extension activity will help students make connections between the concrete (the soccer field) and the abstract (any triangle).

Benchmarks

Research on the Dimensions of a High School Soccer Field

(Typical Size, but Varies):



Calculations:

Since a soccer field is rectangular, each of the interior angles measure 90 degrees and are right angles. Drawing a diagonal, creates two right triangles. When comparing the two paths ran by the soccer players, it is necessary to calculate the distance around the entire rectangular field and to calculate how long the diagonal distance is and multiply that by two since 1 lap = going from one corner to the opposite corner twice. To find the distance around the field, we must find the perimeter (360 + 160 + 360 + 160 or P = 2l + 2w = 2 (360) + 2 (160) = 1040 feet. To find the diagonal distance, we must use the Pythagorean Theorem, which is $a^2 + b^2 = c^2$. This leads to $160^2 + 360^2 = c^2$. Simplifying gives 25,600 + 129,600 = c^2 ; 155,200 = c^2; Taking the square root of both sides leads to c being approximately 393.95431 feet. But, we have to multiply this by 2 to get a full lap distance, which is approximately 787.90862 ft.

It is clear from this information that if we run 20 laps, the shorter path would be the diagonal distance. The total distance ran for 20 laps around the field would be 1040 feet * 20 = 20,800 ft or approximately 3.94 miles since 5,280 feet = 1 mile. On the other hand, the total distance ran using the diagonal path would be 789.90862 feet * 20, which is approximately

15,798.17 feet, which is about 2.99 miles. By using the diagonal path, the soccer players would almost run a mile lesser than the other players.

If a student wants to get in better shape, he or she would choose the longer path. On the other hand, a student who doesn't want to run as much, would choose the shorter path.

Another Option for Determining the Shorter Path:

Since the soccer field is rectangular and the diagonal splits it up into 2 congruent triangles (specifically right), we can use the fact that the sum of the lengths of two sides of a triangle will always be greater than the length of the third side. So, a ½ outside lap would be the 2 sides (legs) of the right triangle and a ½ inside lap would be the 3rd side (hypotenuse). Therefore, the outside lap distance would always be greater than the inside distance. It is not totally necessary to know the actual dimensions of the soccer field to determine which path would be the shortest.

Name: _____

Date: _____

Geometry Teacher: _____

Key:

Follow-up / Extension Activities (*Each Student Completes This!*)

Post-Presentation Questions *(You Must Justify Your Answers!)*

Sample Questions:

1. What are the dimensions of our school soccer field? How do you know?

   The dimensions vary, but the typical size is 120 yards by 55 1/3 yards. I know because I researched this on the Internet (cite source(s)).

2. Do the dimensions of the soccer field matter in this problem? If not, why?

   The dimensions of the soccer field do not really matter in this problem because the sum of the lengths of two sides of a triangle will always be greater than the length of the third side. This is also known as the Triangle Inequality Theorem. The diagonal of the rectangular soccer field splits it into two congruent right triangles. So, we were to consider the distance of a ½ lap, then the two legs of one right triangle would be ½ the distance of one lap around the outside of the soccer field and the diagonal (hypotenuse) would represent ½ the distance of one lap on the "inside" of the soccer field. Since the sum of the two legs will always be greater than the length of the hypotenuse, then it is clear that the shortest path would be along the diagonal. However, we do need to know the dimensions of the soccer field to answer additional questions that our teacher has given us in this task.

3. If you normally run 10 laps in 25 minutes, how long will it take you to run 20 laps?

   50 minutes (set up a proportion)

4. How far (in yards, feet, or meters) are you running each minute on average?

10 laps around the field = 10,400 feet in 25 minutes. Dividing each by 25 gives 416 feet per minute.

5. What is your average running speed?

   To give the speed in miles per hour, I first multiply 416 feet by 60 to determine how many feet I would run in an hour. This gives 24,960 feet per hour. Taking 24,960 and dividing it by 5,280 gives approximately 4.73 miles per hour.

6. If you maintain this pace, will it take you longer to run the 20 laps around the soccer field or 20 laps from corner to corner? How much longer?

   It would take me longer to run 20 laps around the soccer field because it is a longer distance. 20 laps around the field = 20,800 feet and 20 laps along the diagonal is approximately 15,798.17 feet. So, if I run 416 feet per minute, I would set up two different proportions: (416 ft / 1 min. = 20,800 ft / x min.) and (416 ft / 1 min. = 15,798.17 ft / x min.). Doing this leads to it taking 50 minutes to run 20 laps around the field, as mentioned previously, and it taking approximately 37.98 min. to run 20 laps along the diagonal. The difference between the 2 is 12.02 minutes. This is a great break!

7. So, is it further to run 20 laps around the outside of the soccer field or 20 laps from corner to corner (along the diagonal)? How much further?

   The total distance ran for 20 laps around the field would be 1040 feet * 20 = 20,800 ft or approximately 3.94 miles since 5,280 feet = 1 mile. On the other hand, the total distance ran using the diagonal path would be 789.90862 feet * 20, which is approximately 15,798.17 feet, which is about 2.99 miles. By using the diagonal path, the soccer players would almost run a mile lesser than the other players (0.95 miles less). So, it is further to run around the soccer field. Those who choose this path, will run 0.95 miles further.

8. If the coach really wants some players to run "outside" laps and other players to run "inside" laps, calculate how many fewer "outside" laps should be run so that everyone finishes at roughly the same time.

   Since the “inside” lap runners spend approximately 38 minutes running their 20 laps, the “outside” lap runners would need to finish their laps in roughly the same amount of time. So, if it takes the “outside” lap runners 25 minutes to run 10 laps, I set up a proportion to find out how many laps these runners could run

in 38 minutes. After finding out the number of laps, I subtracted this from 20 to determine how many fewer laps the "outside" lap runners would have to run so that every player could finish at about the same time. Here is my work:

$$\frac{25\ minutes}{10\ laps} = \frac{38\ minutes}{x\ laps};$$

25x = 380 (Cross-Multiplication)

x = 15.2 or 15 1/5 laps (Division by 25)

But, it may be a little difficult for the coach and the players to actually determine what 1/5 of a lap is, so it would be easier for the "outside" runners to run 15.5 or 15 ½ laps around the field. In this case, it would take the runners 38.75 minutes or 45 seconds longer to run, but this is only fair since these runners chose the longest path ☺.

$$\frac{25\ minutes}{10\ laps} = \frac{x\ minutes}{15.5\ laps};$$

10x = 387.5 (Cross-Multiplication)

X = 38.75 minutes (Division by 10)

So, the "outside" runners would have to run 4.5 fewer laps than the "inside" runners in order for the players could finish at roughly the same time.

9. Determine if the coach is spending too much time on having the players run 20 laps instead of having them practice soccer drills. If the soccer team normally practices for 90 minutes with a 5 minute water break included, what percent of practice is spent running these 20 laps? Make sure you calculate the percentages for both paths.

   The 5-minute water break is extraneous information since the water break is included in the practice time. So, if it takes the outside runners 50 minutes to complete the 20 laps, approximately 55.6 % of practice time would be dedicated to running. If it takes the inside runners approximately 38 minutes to complete their 20 laps, then approximately 42.2 % of practice time would be dedicated to running. Being able to run for long lengths of time is very important in the game

of soccer. Athletes have to be able to keep up with or run faster than their opponents. Soccer players have to be able to run long periods of time during games. A typical high soccer game has two halves which last 40 minutes each. So, if the team were short of players at a game, they would have developed the endurance to run at least 38 minutes without stopping given that this is what would have been practiced. Soccer drills are also important and the coach still allows time for this, as well.